The Friend of Your Trend

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Abstract

Standard performance attribution to beta and alpha is not simple without full transparency into the investment process. Instead of using ex-post realized returns-based style analysis, this article develops an analytical framework to shed light on ex-ante stylized characteristics of a simple trend following strategy. Our analytical results show that rewards from the trend following strategy embed different degrees of underlying asset beta, which are determined by the asset's return-to-volatility ratio, in addition to the trending behaviors that the strategy is built to harvest. We discuss some practical implications of our results with respect to fees and allocations to trend following strategies.

Keywords— Trend, CTA, Bias, Alpha, Beta

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1 Introduction

Structural positions, such as prolonged overweights of stocks to bonds, embedded in tactical asset allocation (TAA) are analyzed in Lee (2000). Alpha of TAA is decomposed into "alpha due to bias" as a result of structural bets, and "alpha due to volatility capture," which is more related to true investment skill based on successful forecasting. Lee (2000) derives analytical results in understanding how structural bias may explain observed performance of investment strategies, and explores remedies based on filtering out the effects of structural bias over time. [1] By now, a number of studies have documented that many hedge fund strategies have traditional asset class risk premia embedded in their performance. Asness, Krail, and Liew (2001) investigates to what extent hedge funds hedge, and how investors may reconsider performance fees that are driven, in part, by structural biases to traditional risk premia. DaSilva, Lee, and Pornrojnangkool (2009) shows that straight application of the Black-Litterman model in active portfolio management can lead to biases to overweight risky assets.

Trend following as an investment strategy has existed for a long time. Some hedge funds, such as managed futures and commodity trading advisors (CTAs), are believed to use trend following as their predominant investment approaches. While exact calibrations of the strategy, like sizing and risk taking, vary among investors, all trend following strategies, at their root, involve taking long positions in assets that have recently gone up in price and short positions in assets that have gone down.

Strategies built on price trends of individual assets are often said to be trading time-series momentum, while those built on relative asset return trends are trading cross-sectional momentum; see Goyal and Jegadeesh (2018). Cross-sectional momentum trades can be applied to relative trends among assets within the same asset class, such as between stock market indexes or sectors, or among different asset classes, such as equity, credit, and government bonds, for example.

Since the Global Financial Crisis (GFC) of 2008, the investment community has witnessed explosive growth of interest in tail risks hedging strategies. Because direct hedging of tail risks can cause a meaningful performance drag in the long term, some alternative approaches that are expected to produce symmetric payoffs, regardless of asset price direction, are receiving renewed interests. Trend following strategies are believed to have such performance characteristics; see Moskowitz, Ooi, and Pedersen (2012), Asvanunt, Nielsen, and Villalon (2015), Hurst, Ooi, and Pedersen (2017), for examples. As long as the strategy successfully captures both up and down trends in the price of the underlying asset, the strategy can deliver a performance profile with a relatively symmetric payoff when future price volatility of the underlying asset is sufficient. [2]

In a comprehensive study of time series momentum applied to a universe of asset classes, Moskowitz, Ooi, and Pedersen (2012) finds that the existence of positive serial correlations in returns is the key to profitability of trend following strategies, rather than the exact definition of the trading signal. Given its noticeable diversifying role within a core portfolio of traditional risk premia, it is important to understand the key elements that drive a trend following strategy's performance

1. The head of asset allocation at one of the largest fund management companies was interviewed and quoted in a February 8, 2017 interview, "For the first time since the early 2000s we decided to slightly tactically underweight stocks versus bonds ... That actually has helped us over the long run." It is unclear to what extent the position was tactical versus structural.

2. Studies show that the payoff of trend following strategies is determined by going long long-term volatility and short short-term volatility, rather than long the asset volatility.

characteristics, based upon which fair fees may be determined. In this study, we are particularly interested in the role of traditional risk premia, also known as asset betas, in determining the performance characteristics of trend following strategies. Subsequent studies revisited performance of the time-series momentum strategies studied in Moskowitz, Ooi, and Pedersen (2012). Kim, Tse, and Wald (2016) argues that the performance of the strategies is largely a result of volatility-scaling asset risk premia, while Huang et al. (2020) provides evidence that performance is likely driven by historical asset risk premia rather than returns predictability due to the presence of serial correlations.

To motivate our discussions, consider the scenario that the expected return of an asset is a positive constant without serial correlation. Its realized return in a period would simply be the sum of the constant and a random noise. Given the positive asset risk premium, on average, we would expect the trading signal to be positive as well. In this case, the corresponding trend following strategy is expected to deliver positive returns even in the absence of serial correlations, merely because of its structural long bias to the asset. Because the strategy's performance is essentially driven by taking an average long position on an asset that has positive risk premium, one may think of it as something more akin to strategic asset allocation, rather than an active investment strategy based solely on investment skill. Of course, it is more likely that the investment opportunity set is stochastic, so that the asset return dynamics in reality are time-varying and more complicated than the constant expected return example above. It is precisely because of the existence of more complicated returns dynamics that forensic decomposition of a trend following strategy's performance into traditional risk premium and serial correlation components becomes very difficult, if not impossible.

To give a more realistic example, consider a trend following strategy on a single stock index. With a positive equity risk premium, the stock index has a positive drift over time. Over a long enough period, the trend following strategy would, on average, take a positive position on the index. Since the average future return of the index is positive, at least part of the trend following strategy's returns can be attributed to its normal long exposure to the asset risk premium, independent of any serial correlation. It follows that pairwise relative trend following strategies that are applied to assets with different risks so that one asset commands a positive risk premium to the other, such as stocks relative to bonds, low grade credit versus high grade credit, credit versus government bond, and emerging market versus developed market, among others, can also be subject to structural biases.

In this paper, we first present a simulation example of how an information-less strategy could still deliver positive performance. We then provide an analytical framework to understand the characteristics of a trend following strategy, including its positions, performance statistics, and beta and alpha decomposition, as determined by the degree of trending behavior. In summary, we find that, even in the absence of trending behavior in the underlying assets, trend following strategies can still deliver positive performance as long as assets have positive average returns over time. Different degrees of asset beta are embedded in trend performance. Knowing the impact of these structural exposures can help determine the portion of strategy performance that is due to skill. Further applying our framework, we perform a causal analysis of the performance of a CTA index through our analytical lens. Practical implications on uncovering betas from real portfolios, fees, and allocation to trend following strategies are discussed.

2 Informationless Strategy Simulation

To illustrate how asset betas can be embedded in performance and fees, we study a hypothetical case in which an investor attempts to predict future asset returns based on forecasts from regressions. We simulate a hypothetical investment strategy built on a universe of global assets that includes stock futures contract returns of Australia, Canada, Hong Kong, Europe, Japan, UK, US, and MSCI emerging markets in U.S. dollar; 10-year government bond futures contract returns of Australia, Canada, Europe, Japan, UK, and US; and currencies of Australia, Canada, Europe, Japan, Switzerland, and UK versus the US dollar. For the sample period from January 2005 to December 2019, we compare the simulated strategy to the HFRX Macro: Systematic Diversified CTA Index which includes strategies that are typically quantitative in nature in their attempt to benefit from trending or momentum behavior in the underlying liquid asset returns. All data are from Bloomberg.

Each month, the forecast return of each asset is computed based on a rolling 36-month univariate linear regression. We analyze two cases. In the case labeled as OneX, there is only an intercept term and no signal is included in the regression. Clearly, the forecast return is simply the intercept term which captures the historical average return of the asset in the past 36 months. In the second case labeled TwoX, in addition to the intercept term, we introduce one variable which is random noise. This corresponds to a case in which the investor is unaware that the variable is just noise that has no informational value regarding future returns. The forecast return, therefore, is a noisy estimate of the historical average asset return in the last 36 months. The forecast returns of all assets in the universe are then used as inputs to an unconstrained mean-variance optimization using the trailing 36 months to estimate the asset covariance matrix. [3] Positions in the OneX and TwoX strategies are then scaled to target a constant expected annualized 10% volatility each month.

Table 1: Simulated OneX, TwoX, and HFRX Macro: Systematic Diversified CTA Index: 2005:1 - 2019:12

Strategy	Return	Volatility	Return-To-Volatility
OneX	6.08%	8.39%	0.73
TwoX	5.56%	7.91%	0.70
CTA	3.18%	9.25%	0.34

As reported in Table 1, the gross-of-fees and transaction costs return-to-volatility ratios of the informationless strategies OneX and TwoX of 0.73 and 0.70 stand out against the return-to-volatility ratio of 0.34 of the HFRX Macro: Systematic Diversified CTA Index over the same period. For a hedge fund fee of 1% of assets plus a 20% performance fee, these information-less strategies could have earned total fees of over 2%.

Table 2 reports the correlations among the simulated strategies and the CTA index. As expected, OneX and TwoX are highly correlated at 0.94, since the latter is a noisy proxy of the former. Of particular interest is the fact that the CTA index is positively correlated to these informationless strategies at greater than 0.20. Regressions of the CTA index return on the simulated strategy

3. For the first 36 months of the simulation, we use the covariance matrix estimated with look ahead bias of the same period.

Table 2: Correlation Matrix of Simulated OneX, TwoX, and HFRX Macro: Systematic Diversified CTA Index: 2005:1 - 2019:12

	OneX	TwoX	CTA
OneX	1.00		
TwoX	0.94	1.00	
CTA	0.22	0.24	1.00

returns of OneX and TwoX have R-Squares of 4.85% and 5.59%, respectively. This may reflect the fact that the CTA index return is driven by historical returns of the underlying assets to some degree. A later section provides more examples on embedded betas of global equities and global bonds in the CTA index versus the analytical predictions of our simplified model to be discussed.

3 Trend Following Strategies With and Without Serial Correlations

In this section, we provide a more formal analysis of embedded betas in trend following strategies. Levine and Pedersen (2016) generalizes a number of trend indicators to reveal that they are equivalent representations in their most general forms. In order to keep our discussions tractable, we follow Moskowitz, Ooi, and Pedersen (2012) in defining a generic trend following strategy that takes positions, either long or short, based on the realized recent return of the asset. Specifically, at the end of the period t, our next-period-intended-position s_t in the underlying asset will depend on the last observed sign of the asset excess return, r_t,[4]

$$s_t := \mathbf{sign}(r_t) \tag{1}$$

Therefore, the strategy return z_{t+1} at the end of period $t+1$, can be represented as

$$z_{t+1} = s_t r_{t+1} \tag{2}$$

We are interested in the expected strategy return, $\mu_z = \mathbf{E}[z_t]$, the risk as measured by the volatility of the strategy return, $\sigma_z = \sqrt{\mathbf{Var}[z_t]}$, its expected loading on the underlying asset, $\beta_z = \mathbf{E}[s_t]$, and its propensity to take a long position in the underlying asset $\mathbf{Pr}[s_t > 0]$. Given these statistics, we will be able to compute the expected alpha of the strategy, which is defined as the asset-beta adjusted return as follows

$$\mathbf{E}[\alpha_z] = \mu_z - \beta_z \mu \tag{3}$$

In the appendix, we derive general performance characteristics of the generic trend following strategy in which asset returns follow a first order autoregressive, AR(1), process with positive serial correlation. The case of constant growth, which serves as the base of reference can be interpreted

4. Christoffersen and Diebold (2006) illustrates that dependence in conditional mean, signs of asset returns, and asset return volatilities are interrelated. Specifically, serial dependence in signs of asset returns can be a result of the serial dependence in asset return volatilities even in the absence of serial dependence in conditional mean.

as a special case when the first order serial correlation is zero. The characteristics of the trend following strategy in the absence of any serial correlation sheds light on what performance one may expect as merely a result of long bias to the asset risk premium. Separately, we also analyze the case of asset returns following a first order moving average, MA(1), process. Since the performance characteristics are identical to the case of AR(1), we do not report the results here.

We assume that the returns are generated by the following first order autoregressive AR(1) model:

$$r_t = c + \phi r_{t-1} + \sigma_\varepsilon \varepsilon_t, \quad \varepsilon_t \sim \mathcal{N}(0, 1), \quad |\phi| < 1 \tag{4}$$

For a realistic but simple model of the trend in asset returns, we assume $\phi \geq 0$ and without loss of generality, we restrict $c \geq 0$ as it is sensible that assets should have positive expected returns in the long run. Let h denote the lag for the autocorrelation function $\mathbf{acf}[r_t, h] := \rho_h$. It can be shown that r_t is stationary and is normally distributed with:

$$\begin{aligned} \mu = \mathbf{E}[r_t] &= \frac{c}{1-\phi} \geq 0 \\ \sigma^2 = \mathbf{Var}[r_t] &= \frac{\sigma_\varepsilon^2}{1-\phi^2} \\ \rho_h &= \phi^h, h \in \mathbb{N} \end{aligned} \tag{5}$$

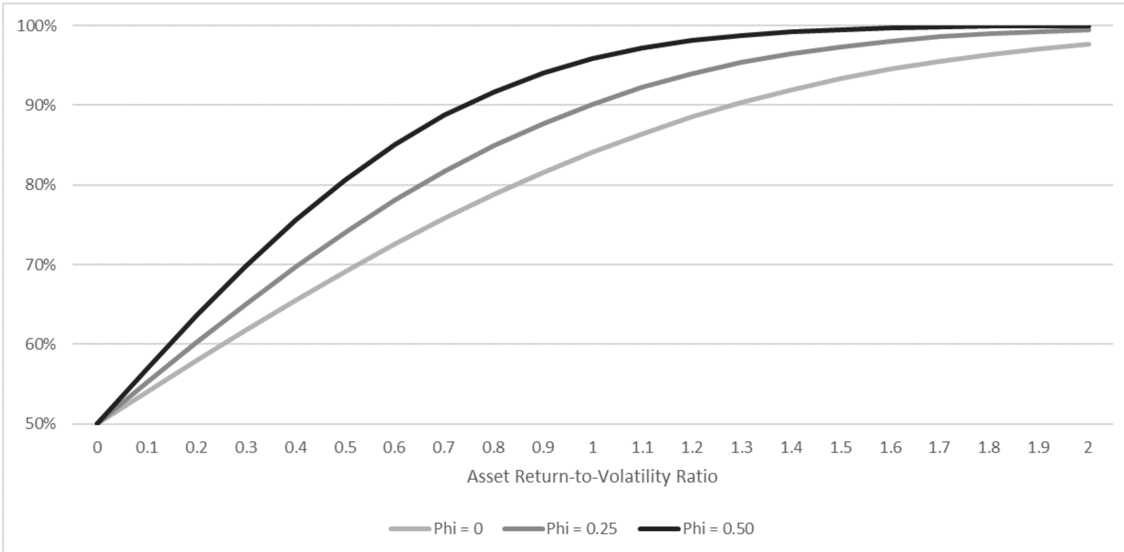
Denote the probability density function and the cumulative density function of the standard normal distribution as φ and Φ. Therefore,

$$\begin{aligned} \mathbf{E}[s_t] = \mathbf{E}[\mathbf{sign}(r_t)] &= 2\,\Phi\left(\frac{\mu}{\sigma}\right) - 1 \\ &= 2\,\Phi\left(\frac{c}{\sigma_\varepsilon}\sqrt{\frac{1+\phi}{1-\phi}}\right) - 1 \end{aligned} \tag{6}$$

$$\begin{aligned} \mathbf{Pr}[s_t > 0] = \mathbf{Pr}[\mathbf{sign}(r_t) > 0] &= \Phi\left(\frac{\mu}{\sigma}\right) \\ &= \Phi\left(\frac{c}{\sigma_\varepsilon}\sqrt{\frac{1+\phi}{1-\phi}}\right) \end{aligned} \tag{7}$$

Note that as long as the asset has a positive expected return as assumed in the model, the trend strategy is expected to take a long position in the asset. Likewise, the probability of going long the asset is higher than 50%. To visualize the relationship, Figure 1 plots the probability of going long the asset as a function of the asset's return-to-volatility ratio based on a first order autocorrelation coefficient at 0, 0.25, and 0.5, respectively. The case of zero autocorrelation corresponds to the case of constant growth. Clearly, the probability of going long the asset increases with the return-to-volatility ratio of the asset through the standard normal cumulative density function. By now, we have established the result that the trend following strategy is biased to go long the underlying asset even when there is no trending behavior.

Figure 1: Probability of Long Asset



3.1 Performance Characteristics

We list below the key performance characteristics of the strategy. Detailed derivations are provided in the appendix.

$$\mu_z := \mathbf{E}[z_t] = \mu \left(2\,\Phi\Big(\frac{\mu}{\sigma}\Big) - 1\right) + 2\phi\sigma\,\varphi\Big(\frac{\mu}{\sigma}\Big) \tag{8}$$

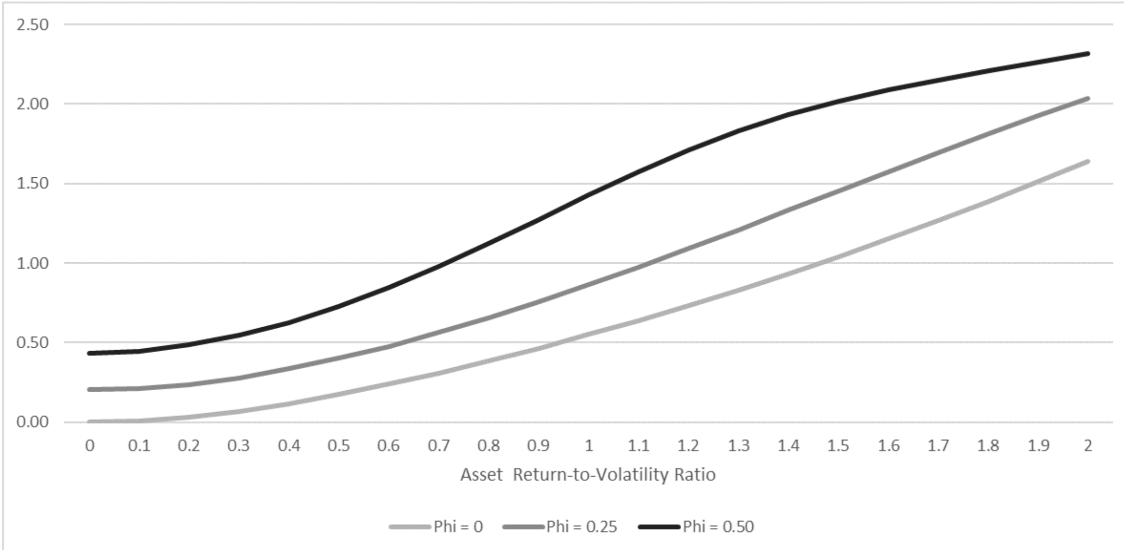
$$\begin{aligned} \sigma_z^2 := \mathbf{Var}[z_t] = \sigma^2 &+ 4\mu^2\,\Phi\Big(\frac{\mu}{\sigma}\Big)\left(1 - \Phi\Big(\frac{\mu}{\sigma}\Big)\right) \\ &- 4\phi\sigma\,\varphi\Big(\frac{\mu}{\sigma}\Big)\left(\phi\sigma\,\varphi\Big(\frac{\mu}{\sigma}\Big) + \mu\left(2\,\Phi\Big(\frac{\mu}{\sigma}\Big) - 1\right)\right) \end{aligned} \tag{9}$$

For illustration, we set the asset return volatility at 10%; that is, $\sigma = 10\%$. Figure 2 plots the resulting return-to-volatility ratio of the trend following strategy as a function of other parameters.

In the way we model the asset return dynamics, the degree of trending behavior is reflected by the AR(1) coefficient. It is intuitive to see that, everything else equal, the risk-adjusted reward of the trend following strategy rises with the AR(1) coefficient. However, why does the risk-adjusted reward of the strategy also rise with the asset's return-to-volatility ratio? More interesting, in the case of constant growth with $\phi = 0$ with no trending behavior in the asset, why does the trend following strategy deliver a positive risk-adjusted reward? As it turns out, the asset's return-to-volatility ratio is the friend of the trend - a very good friend indeed.

The case of constant growth with no serial correlation in an asset with zero expected return sets the benchmark for discussions. In such a scenario, the probability of going long the asset and expected strategy return are both zero as expected and shown in Figure 1 and Figure 2, respectively. To isolate the effect of trending on performance, we continue to fix the asset return-to-volatility at zero. When the AR(1) coefficient is raised to 0.25 and 0.50, the strategy return-to-volatility ratio improves to 0.20 and 0.44, respectively. Note that these improvements are entirely the results of stronger trending behavior, which is precisely what a trend following strategy is built to capture.

Figure 2: Trend Following Strategy Return-To-Volatility Ratio



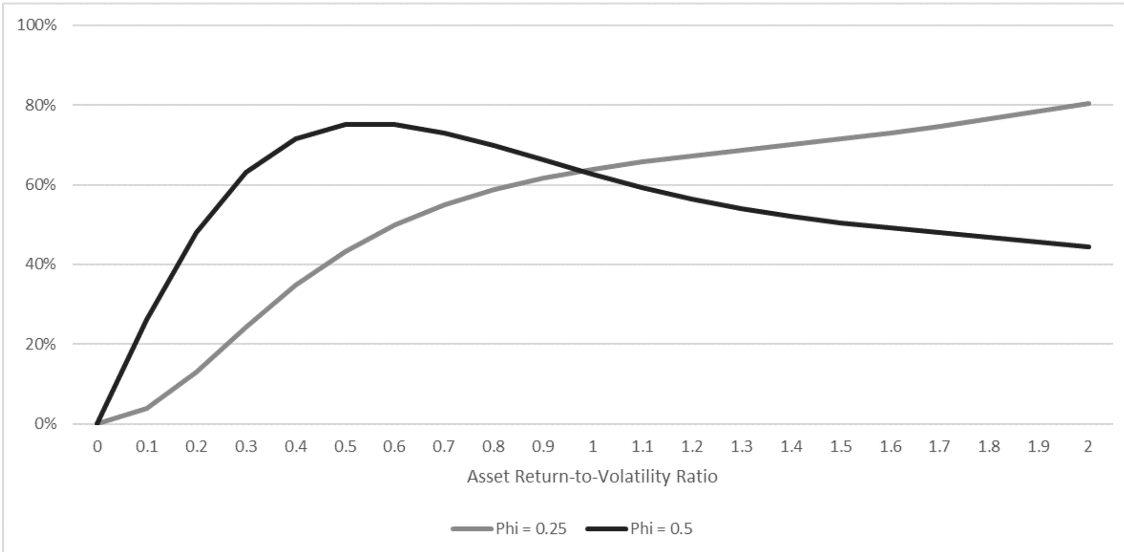
Next, we analyze scenarios where the return-to-volatility ratio of the asset ranges from zero to as high as 2. Professional investment experience suggests that long-term asset return-to-volatility ratios should fall within a range of 0.2 to 0.6. To understand why an asset with no trending behavior would still give rise to a positive trend strategy return, we first note that, given the specifications of the model in the study, the trend following strategy volatility is quite close to the asset volatility and is relatively stable regardless of what other parameter values we set. We therefore shift our attention to the two terms in the trend strategy expected return in equation (8).

The second term includes the AR(1) coefficient, ϕ, through which the strength of trending behavior is captured. As expected, the stronger the first order serial correlation in asset return, the higher the strategy expected return. In addition, the trend strategy will also benefit from the higher volatility and return-to-volatility ratio of the asset, but if, and only if, the asset shows trending behavior through and amplified by the AR(1) coefficient, ϕ.

Perhaps most interesting, the first term has nothing to do with asset trending behavior and is driven entirely by the asset's long-term performance. As discussed earlier, a positive asset return suffices to guarantee that the first term is positive. By now, we have shown that the trend strategy has embedded asset beta regardless of whether there is trending behavior or not. We next investigate the relative importance of the two terms in determining the return-to-volatility ratio of the trend strategy.

Figure 3 plots the portion of the trend following strategy return-to-volatility ratio that is attributable to the asset beta. This is the ratio of the trend following strategy's return-to-volatility ratio when asset returns follow constant growth with $\phi = 0$ to the strategy's return-to-volatility ratio when asset returns have different degrees of serial correlations. It is interesting to note that, given a reasonable range of asset return-to-volatility ratios, as much as 75% of the trend strategy risk-adjusted return can be attributed to the asset beta and has nothing to do with the degree of trending.

Figure 3: Attribution of Trend Following Strategy Return-To-Volatility Ratio to Asset Beta



3.2 Beta and Alpha

The presence of asset beta implies that not all reward from the trend following strategy are related to the skill, or alpha, of capturing the trending behavior of the asset returns. We proceed to analyze the asset beta and alpha. Again, detailed derivations are provided in the appendix.

$$\beta_z = \left(2\,\Phi\Big(\frac{\mu}{\sigma}\Big) - 1\right) + 2\phi(1-\phi)\frac{\mu}{\sigma}\,\varphi\Big(\frac{\mu}{\sigma}\Big) \tag{10}$$

$$\begin{aligned} \alpha_z &:= \mu_z - \beta_z \mu \\ &= 2\frac{\phi}{\sigma}\,\varphi\Big(\frac{\mu}{\sigma}\Big)\left(1 - (1-\phi)\,\frac{\mu^2}{\sigma^2}\right) \end{aligned} \tag{11}$$

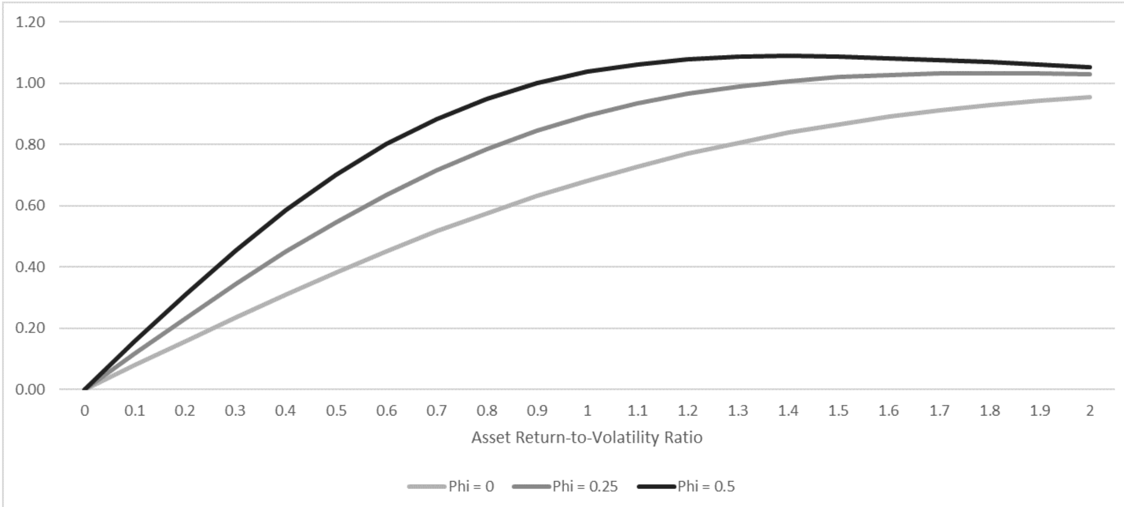
Therefore,

$$\alpha_z > 0 \iff \phi > 1 - \frac{1}{(\mu/\sigma)^2} \tag{12}$$

Similar to the strategy expected return, there are two terms in the beta. The first term is guaranteed to be greater than zero as long as the asset has positive expected return and is independent of the trending behavior of the asset. The second term determines the portion of beta that is a result of the serial correlation in asset returns. Note that the asset's return-to-volatility ratio lifts the beta through both terms.

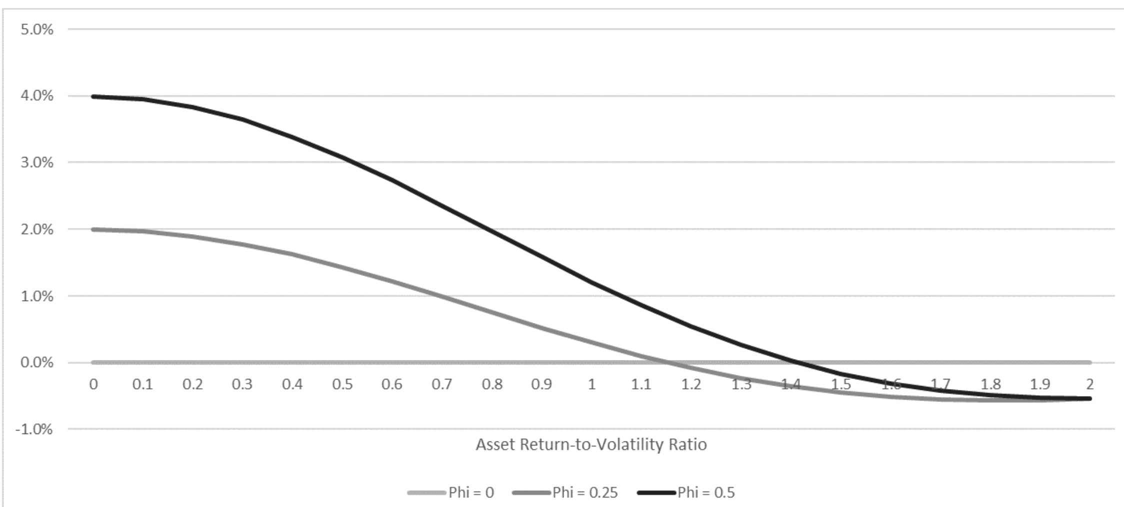
In Figure 4, it is shown that the trend following strategy's asset beta increases with the asset's return-to-volatility ratio and strength of trending as measured by the AR(1) coefficient. Starting with constant growth and no trending behavior again, we observe that the beta can be higher than 0.50 when the asset's return-to-volatility ratio is around 0.70 - this is high, but not impossible.

Figure 4: Trend Following Strategy Asset Beta



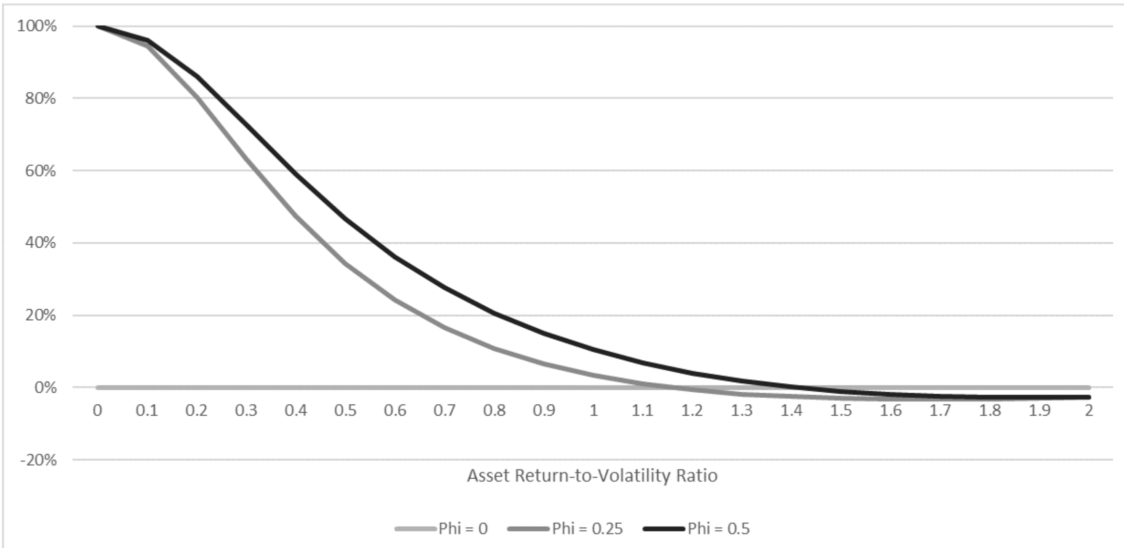
In the absence of trending behavior, the beta never exceeds 1.00, however. Because of the serial correlation in asset returns, beta can slightly exceed 1.00 given, in our experience, unrealistically high asset return-to-volatility ratios of 0.80 or above, for example.

Figure 5: Trend Following Strategy Alpha



As for alpha, it is intuitive to see in Figure 5 that the alpha is zero when there is no serial correlation in asset returns. Similar to other performance statistics, alpha increases with the degree of trending as measured by the AR(1) coefficient ϕ. Perhaps more interesting is the fact that alpha is a monotonically decreasing function of the asset's return-to-volatility ratio. The higher the asset's return-to-volatility ratio, the lower the alpha, which is defined as the beta-adjusted return of the strategy. Alpha becomes negative at sufficiently high return-to-volatility ratios. This is because beta increases faster than alpha as the asset's return-to-volatility ratio rises. Equation (12) shows that the AR(1) coefficient must be high enough with respect to a given asset's return-to-volatility ratio for alpha to be positive. Our analytical results on alpha resonate well with the empirical findings of Baltas and Ivanova (2019a), which concludes that timing strategies of assets with higher mean-to-volatility ratios require substantially higher manager skill to perform respectably well on a buy-and-hold basis.

Figure 6: Alpha As Percentage of Trend Following Strategy Return



Like Figure 3, we also look into alpha as a percentage of the trend strategy return in different scenarios of trending behavior. We can see in Figure 6 that as the asset return-to-volatility ratio rises, the portion of the trend following strategy return attributable to alpha declines until alpha turns slightly negative.

4 Application Example: CTA

To illustrate application of our analytical framework, we perform a return-based style analysis of a CTA index. The HFRX Macro: Systematic Diversified CTA Index includes strategies that are typically quantitative in nature and that attempt to benefit from trending or momentum behavior in the underlying liquid asset returns. It is highly likely that the strategies trade multiple global asset futures contracts. For tractability, our analytical framework applies to the trend following strategy of single asset only, and arguably is not capable of reflecting the multi-asset nature of the index constituents. Nevertheless, our casual analysis of the index helps call attention to the need for a more formal framework to understand how underlying asset betas may drive the index performance. To keep the analysis simple and yet insightful, we regress the CTA index excess returns on excess returns of the MSCI ACWI index and the Barclays Capital Global Aggregate index only. We use the 3-month T-bill as the proxy for cash return. All monthly returns data are collected from Bloomberg from January 2005 through December 2019. The performance statistics of the underlying assets and CTA, as well as the return-based style regression analysis results are summarized in Table 3 and Table 4 below.

Table 3: Performance of MSCI ACWI Index, Barclays Capital Global Aggregate Index, and HFRX Macro: Systematic CTA Index: 2005:1 - 2019:12

	ACWI	Global Agg	CTA
Annualized Return	7.53%	4.31%	3.18%
Annualized Volatility	13.09%	2.63%	9.25%
Return-To-Volatility	0.58	1.64	0.34

Table 4: Style Analysis of HFRX Macro: Systematic CTA Index Excess Returns: 2005:1 - 2019:12

	Intercept	ACWI	Global Agg
Coeff	0.00	-0.07	0.67
t-stat	0.32	-1.45	2.62**
p-value	75.1%	14.8%	1.0%
Adj. R-Square	4.00%		

Note: ** and * denote statistical significance at 1% and 5%, respectively.

Over the sample period, both the ACWI and Global Agg indices have delivered high return-to-volatility ratios at 0.58 and 1.64, respectively. The CTA index had a return of 3.18% and a volatility of 9.25%, which gives a 0.34 return-to-volatility ratio. Based on the generic definition of the trend following strategy in equation (1), the strategy's betas with respect to the ACWI and Global Agg according to equation (10), given their return-to-volatility ratios for the case of zero correlations in their monthly returns, are about 0.44 and 0.90, respectively.

The ACWI beta is estimated to be -0.07, substantially different from that predicted by the the analytical results. Among the many possible reasons behind the discrepancy, it might reflect that the CTA index did not take much equity risk. The Global Agg beta of 0.67 is closer to the analytical result of 0.90, however. [5] A later section provides more discussion on this matter.

5 Practical Implications

In this section, we illustrate some practical ways to apply the analytical results. In particular, we would like to understand the implications on management fees and allocations to trend following strategies. Note that our results remain valid with respect to cross-asset trend following strategies as long as there is a dispersion of risks across assets in the strategy. Relative trends among asset classes, such as between stock and bond returns, or within asset classes, such as between high yield and investment grade bonds returns, can give rise to a bias of going long the riskier asset and short the less risky asset.

5. We also follow Scholes and Williams (1977) to include three lags of explanatory variables in the regression analysis. The results are similar to those reported.

5.1 Fees

CTAs are generally trend following hedge funds. For instance, there are CTA hedge fund indices provided by hedge fund research companies. In the hedge fund industry, it is standard practice to charge both a management fee based on a fixed percentage of assets under management and a performance fee that is a portion of the strategy return in excess of a pre-defined hurdle rate of return, which can be zero, a cash return, or other structures. Given the existence of asset beta in trend following strategies, part of their performance, and therefore their performance fees, is the result of asset returns that have nothing to do with skill, but instead are entirely driven by passive exposure to the underlying asset. In the over-simplified case of a zero hurdle rate, the portion of the performance fee attributable to alpha is identical to the portion of the trend following strategy return attributable to alpha as shown in Figure 6. Within a reasonable range of asset return-to-volatility ratios, about 20% to 70% of the performance fee can be attributed to the asset beta according to Figure 6. In the case of constant growth without trending, 100% of the performance fee is attributable to the asset beta.

5.2 Allocation to Trend Following Strategies

A number of studies have positioned trend following strategies as diversification strategies that have convex payoff profile with respect to their underlying assets; see Moskowitz, Ooi, and Pedersen (2012) for example. In the industry, some consider long volatility as a characteristic of these strategies, possibly based on its somewhat convex payoff profile. According to our analytical results, alpha should be a result of trending behavior as determined by the serial correlations of the asset returns. It is well known that in the existence of positive (negative) serial correlations, long-term volatility is higher (lower) than the short-term volatility. In other words, the alpha of a trend (contrarian) strategy is more accurately described as the result of long (short) long-term volatility and short (long) short-term volatility. Studies such as Baz et al. (2019) and Harvey et al. (2019) compare and contrast using trend following strategies and others in downside risk hedging. In particular, Baltas and Ivanova (2019b) provides a framework to consider allocation to trend following strategies, proposing tracking error as a metric rather than following traditional optimization approaches which lead to unrealistically large allocations. Clearly, embedded betas of the underlying assets in the trend strategy will also drive its tracking error with respect to the strategic portfolio as well as other relevant performance measures. Our analytical framework and results in this paper add to the list of considerations when determining optimal allocations to trend following strategies.

5.3 Finding Betas

Given the multi-asset nature of most trend following strategies and the stochastic opportunity set with ever changing return dynamics, it may not be as easy to reverse engineer the full extent of structural biases toward risky assets embedded in these strategies. Below, we briefly discuss several factors that can mask these structural biases.

Our derivations assume that the asset return process is ergodic. In reality, it is often observed that volatility tends to be higher when return is negative. If the trend following strategy has a short position while the asset return is negative at higher volatility, it is possible that these profitable short positions can mitigate some of the strategy returns due to the structural bias of going long the asset over the longer sample period. As a result, a regression of the realized strategy returns on the asset returns may not be able to extract the degree of structural bias that is higher than what the regression coefficient may suggest.

The multi-asset, multi-dimensional nature of many trend following strategies can also help make the structural bias look more benign. Consider the case of a universe of multiple assets in which each asset follows its own autoregressive process with different degrees of trending behavior, and the assets are correlated to different degrees. Taking into account the reality that the joint distributions can be time-varying, it is not difficult to see the challenge of finding betas using returns based analysis. Also, risks taken on different assets are likely time-varying. Nevertheless, the clean result of embedding beta in univariate trend strategy returns provides a base of reference.

6 Conclusion

While the concept of beta versus alpha is clear in theory, it is by no means straightforward to separate one from another when evaluating performance without full transparency into the investment process and details of historical positions. This article attempts to use a simplified and tractable analytical framework to derive stylized characteristics of a trend following strategy. Our results suggest that one should expect to have parts of the rewards from the strategy coming entirely from asset beta, which is a function of the asset return efficiency, as measured by return-to-volatility ratio, and of the trending behavior that the strategy is paid to harvest. While we discuss some practical implications of our results on considerations of management fees and allocations to the trend following strategies, we would reassure their role as potential diversifiers to traditional portfolios of asset betas. Their utility, however, may need to be discounted appropriately given the investors' estimates of their embedded betas.

7 Appendix

We derive performance statistics of the trend strategy with an underlying asset following a first order autoregressive process, AR(1). The case of constant growth is a special case in which the first order serial correlation, ϕ, is zero.

To compute the expected strategy return, μ_z, recall that the probability density function of the standard normal distribution is

$$\begin{aligned} \varphi(x) &= \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right) \implies \\ \varphi'(x) &= -x\frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right) \\ &= -x\varphi(x). \end{aligned} \tag{13}$$

Since $r_t \sim \mathcal{N}(\mu, \sigma^2)$, we can express r_t as $r_t = \mu + \sigma e$ with $e \sim \mathcal{N}(0, 1)$. Given

$$\begin{aligned} \mathbf{E}[\mathbf{sign}(\mu + \sigma e)\, e] &= \int_{-\infty}^{\infty} \mathbf{sign}(\mu + \sigma x)\, x\, \varphi(x)\, \mathrm{d}x \\ &= \int_{-\infty}^{-\frac{\mu}{\sigma}} -1x\, \varphi(x)\, \mathrm{d}x + \int_{-\frac{\mu}{\sigma}}^{\infty} 1x\, \varphi(x)\, \mathrm{d}x \\ &= \int_{-\infty}^{-\frac{\mu}{\sigma}} \varphi'(x)\, \mathrm{d}x - \int_{-\frac{\mu}{\sigma}}^{\infty} \varphi'(x)\, \mathrm{d}x \\ &= \varphi(x) \Big|_{x=-\infty}^{x=-\frac{\mu}{\sigma}} - \varphi(x) \Big|_{-\frac{\mu}{\sigma}}^{x=\infty} \\ &= \varphi\Big(-\frac{\mu}{\sigma}\Big) + \varphi\Big(-\frac{\mu}{\sigma}\Big) = 2\,\varphi\Big(-\frac{\mu}{\sigma}\Big) \\ &= 2\,\varphi\Big(\frac{\mu}{\sigma}\Big), \end{aligned} \tag{14}$$

we get

$$\begin{aligned} \mathbf{E}[\mathbf{sign}(r_t)\, r_t] &= \mathbf{E}[\mathbf{sign}(\mu + \sigma e)\, (\mu + \sigma e)] \\ &= \mu\, \mathbf{E}[\mathbf{sign}(\mu + \sigma e)] + \sigma\, \mathbf{E}[\mathbf{sign}(\mu + \sigma e)\, e] \\ &= \mu \left(2\,\Phi\Big(\frac{\mu}{\sigma}\Big) - 1\right) + \sigma\, \mathbf{E}[\mathbf{sign}(\mu + \sigma e)\, e] \\ &= \mu \left(2\,\Phi\Big(\frac{\mu}{\sigma}\Big) - 1\right) + 2\sigma\, \varphi\Big(\frac{\mu}{\sigma}\Big). \end{aligned} \tag{15}$$

Therefore, utilizing (15), we derive that

$$\begin{aligned} \mu_z := \mathbf{E}[z_t] &= \mathbf{E}[s_{t-1} r_t] = \mathbf{E}[\mathbf{sign}(r_{t-1})\, r_t] = \mathbf{E}[\mathbf{sign}(r_{t-1})\, (c + \phi r_{t-1} + \sigma_\varepsilon \varepsilon_t)] \\ &= c\, \mathbf{E}[\mathbf{sign}(r_{t-1})] + \sigma_\varepsilon\, \mathbf{E}[\mathbf{sign}(r_{t-1})\, \varepsilon_t] + \phi\, \mathbf{E}[\mathbf{sign}(r_{t-1})\, r_{t-1}] \\ &= c\, \mathbf{E}[\mathbf{sign}(r_{t-1})] + \sigma_\varepsilon 0 + \phi\, \mathbf{E}[\mathbf{sign}(r_{t-1})\, r_{t-1}] \\ &= c \left(2\,\Phi\Big(\frac{\mu}{\sigma}\Big) - 1\right) + \phi \left(\mu \left(2\,\Phi\Big(\frac{\mu}{\sigma}\Big) - 1\right) + 2\sigma\, \varphi\Big(\frac{\mu}{\sigma}\Big)\right) \\ &= (c + \phi\mu) \left(2\,\Phi\Big(\frac{\mu}{\sigma}\Big) - 1\right) + 2\phi\sigma\, \varphi\Big(\frac{\mu}{\sigma}\Big) \\ &= \mu \left(2\,\Phi\Big(\frac{\mu}{\sigma}\Big) - 1\right) + 2\phi\sigma\, \varphi\Big(\frac{\mu}{\sigma}\Big) \end{aligned} \tag{16}$$

$$
\begin{aligned}
\sigma_z^2 := \mathbf{Var}[z_t] &= \mathbf{E}\big[z_t^2\big] - \mathbf{E}[z_t]^2 = \mathbf{E}\Big[(s_{t-1} r_t)^2\Big] - \mu_z^2 \\
&= \mathbf{E}\big[r_t^2\big] - \mu_z^2 = \big(\mathbf{Var}[r_t] + \mu^2\big) - \mu_z^2 \\
&= \sigma^2 + \mu^2 - \left(\mu\left(2\,\Phi\Big(\frac{\mu}{\sigma}\Big) - 1\right) + 2\phi\sigma\,\varphi\Big(\frac{\mu}{\sigma}\Big)\right)^2 \\
&= \sigma^2 + 4\mu^2\,\Phi\Big(\frac{\mu}{\sigma}\Big)\left(1 - \Phi\Big(\frac{\mu}{\sigma}\Big)\right) \\
&\quad - 4\phi\sigma\,\varphi\Big(\frac{\mu}{\sigma}\Big)\left(\phi\sigma\,\varphi\Big(\frac{\mu}{\sigma}\Big) + \mu\left(2\,\Phi\Big(\frac{\mu}{\sigma}\Big) - 1\right)\right)
\end{aligned}
\tag{17}
$$

$$
\begin{aligned}
\beta_z &:= \frac{\mathbf{Cov}[z_t, r_t]}{\mathbf{Var}[r_t]} = \frac{\mathbf{Cov}[\mathbf{sign}(r_{t-1})\, r_t, r_t]}{\sigma^2} \\
&= \frac{1}{\sigma^2}\left\{\mathbf{E}\big[r_t^2\,\mathbf{sign}(r_{t-1})\big] - \mathbf{E}[r_t]\,\mathbf{E}[\mathbf{sign}(r_{t-1})\, r_t]\right\} \\
&= \frac{1}{\sigma^2}\left\{\mathbf{E}\Big[(c + \phi r_{t-1} + \sigma_\varepsilon \varepsilon_t)^2\,\mathbf{sign}(r_{t-1})\Big] - \mu\left(\mu\left(2\,\Phi\Big(\frac{\mu}{\sigma}\Big) - 1\right) + 2\phi\sigma\,\varphi\Big(\frac{\mu}{\sigma}\Big)\right)\right\} \\
&= \frac{1}{\sigma^2}\Big\{\big(c^2 + \sigma_\varepsilon^2\big)\,\mathbf{E}[\mathbf{sign}(r_{t-1})] + 2c\phi\,\mathbf{E}[r_{t-1}\,\mathbf{sign}(r_{t-1})] + \phi^2\,\mathbf{E}\big[r_{t-1}^2\,\mathbf{sign}(r_{t-1})\big] \\
&\quad - \mu\left(\mu\left(2\,\Phi\Big(\frac{\mu}{\sigma}\Big) - 1\right) + 2\phi\sigma\,\varphi\Big(\frac{\mu}{\sigma}\Big)\right)\Big\}.
\end{aligned}
\tag{18}
$$

$\mathbf{E}[\mathbf{sign}(r_{t-1})] = 2\,\Phi\big(\frac{\mu}{\sigma}\big) - 1$ and $\mathbf{E}[r_{t-1}\,\mathbf{sign}(r_{t-1})] = \mu\,\big(2\,\Phi\big(\frac{\mu}{\sigma}\big) - 1\big) + 2\sigma\,\varphi\big(\frac{\mu}{\sigma}\big)$ are from (6) and (15), respectively. To calculate the term $\mathbf{E}\big[r_{t-1}^2\,\mathbf{sign}(r_{t-1})\big]$, recall that we can express r_t as $r_t = \mu + \sigma e$ where $e \sim \mathcal{N}(0,1)$. Therefore,

$$
\begin{aligned}
\mathbf{E}\big[r_{t-1}^2\,\mathbf{sign}(r_{t-1})\big] &= \mathbf{E}\Big[(\mu + \sigma e)^2\,\mathbf{sign}(\mu + \sigma e)\Big] \\
&= \mu^2\,\mathbf{E}[\mathbf{sign}(\mu + \sigma e)] + 2\mu\sigma\,\mathbf{E}[e\,\mathbf{sign}(\mu + \sigma e)] + \\
&\qquad \sigma^2\,\mathbf{E}\big[e^2\,\mathbf{sign}(\mu + \sigma e)\big] \\
&= \mu^2\left(2\,\Phi\Big(\frac{\mu}{\sigma}\Big) - 1\right) + 2\mu\sigma 2\,\varphi\Big(\frac{\mu}{\sigma}\Big) + \sigma^2\,\mathbf{E}\big[e^2\,\mathbf{sign}(\mu + \sigma e)\big]
\end{aligned}
\tag{19}
$$

From (13) and using integration by parts, we have

$$
\mathbf{E}\big[e^2\,\mathbf{sign}(\mu + \sigma e)\big] = 2\,\Phi\Big(\frac{\mu}{\sigma}\Big) - 1 - 2\frac{\mu}{\sigma}\,\varphi\Big(\frac{\mu}{\sigma}\Big)
\tag{20}
$$

Substituting (20) into (19), we get

$$
\begin{aligned}
\mathbf{E}\big[r_{t-1}^2\,\mathbf{sign}(r_{t-1})\big] &= \mu^2\left(2\,\Phi\Big(\frac{\mu}{\sigma}\Big) - 1\right) + 2\mu\sigma 2\,\varphi\Big(\frac{\mu}{\sigma}\Big) + \sigma^2\left(2\,\Phi\Big(\frac{\mu}{\sigma}\Big) - 1 - 2\frac{\mu}{\sigma}\right) \\
&= \big(\mu^2 + \sigma^2\big)\left(2\,\Phi\Big(\frac{\mu}{\sigma}\Big) - 1\right) + 2\mu\sigma\,\varphi\Big(\frac{\mu}{\sigma}\Big).
\end{aligned}
\tag{21}
$$

Substituting (21) into (18), we get

$$
\begin{aligned}
\beta_z = \frac{1}{\sigma^2}\Big\{&\left(c^2+\sigma_\varepsilon^2\right)\left(2\,\Phi\Big(\frac{\mu}{\sigma}\Big)-1\right)+\\
&2c\phi\left(\mu\left(2\,\Phi\Big(\frac{\mu}{\sigma}\Big)-1\right)+2\sigma\,\varphi\Big(\frac{\mu}{\sigma}\Big)\right)+\\
&\phi^2\left(\left(\mu^2+\sigma^2\right)\left(2\,\Phi\Big(\frac{\mu}{\sigma}\Big)-1\right)+2\mu\sigma\,\varphi\Big(\frac{\mu}{\sigma}\Big)\right)\\
&-\mu\left(\mu\left(2\,\Phi\Big(\frac{\mu}{\sigma}\Big)-1\right)+2\phi\sigma\,\varphi\Big(\frac{\mu}{\sigma}\Big)\right)\Big\}\\
=\frac{1}{\sigma^2}\Big\{&\left(2\,\Phi\Big(\frac{\mu}{\sigma}\Big)-1\right)\left(c^2+\sigma_\varepsilon^2+2c\phi\mu+\phi^2\left(\mu^2+\sigma^2\right)\right)+\\
&4c\phi\sigma\,\varphi\Big(\frac{\mu}{\sigma}\Big)+2\phi^2\mu\sigma\,\varphi\Big(\frac{\mu}{\sigma}\Big)-\mu\left(\mu\left(2\,\Phi\Big(\frac{\mu}{\sigma}\Big)-1\right)+2\phi\sigma\,\varphi\Big(\frac{\mu}{\sigma}\Big)\right)\Big\}\\
=\frac{1}{\sigma^2}\Big\{&\left(2\,\Phi\Big(\frac{\mu}{\sigma}\Big)-1\right)\left(\left(\mu^2+\sigma^2\right)\left(1-\phi^2\right)+\phi^2\left(\mu^2+\sigma^2\right)\right)+\\
&2\mu\sigma\phi\,\varphi\Big(\frac{\mu}{\sigma}\Big)\left(2\left(1-\phi\right)+\phi\right)-\mu\left(\mu\left(2\,\Phi\Big(\frac{\mu}{\sigma}\Big)-1\right)+2\phi\sigma\,\varphi\Big(\frac{\mu}{\sigma}\Big)\right)\Big\}\\
=\frac{1}{\sigma^2}\Big\{&\left(2\,\Phi\Big(\frac{\mu}{\sigma}\Big)-1\right)\left(\mu^2+\sigma^2\right)+2\mu\sigma\phi(2-\phi)\,\varphi\Big(\frac{\mu}{\sigma}\Big)\\
&-\mu\left(\mu\left(2\,\Phi\Big(\frac{\mu}{\sigma}\Big)-1\right)+2\phi\sigma\,\varphi\Big(\frac{\mu}{\sigma}\Big)\right)\Big\}\\
=\frac{1}{\sigma^2}\Big\{&\sigma^2\left(2\,\Phi\Big(\frac{\mu}{\sigma}\Big)-1\right)+2\mu\sigma\phi\,\varphi\Big(\frac{\mu}{\sigma}\Big)\left(2-\phi-1\right)\Big\}\\
=&\left(2\,\Phi\Big(\frac{\mu}{\sigma}\Big)-1\right)+2\phi(1-\phi)\frac{\mu}{\sigma}\,\varphi\Big(\frac{\mu}{\sigma}\Big)
\end{aligned}
\tag{22}
$$

$$
\begin{aligned}
\alpha_z :&= \mu_z-\beta_z\mu\\
&=\mu\left(2\,\Phi\Big(\frac{\mu}{\sigma}\Big)-1\right)+2\phi\sigma\,\varphi\Big(\frac{\mu}{\sigma}\Big)-\left(\left(2\,\Phi\Big(\frac{\mu}{\sigma}\Big)-1\right)+2\phi(1-\phi)\frac{\mu}{\sigma}\,\varphi\Big(\frac{\mu}{\sigma}\Big)\right)\mu\\
&=2\phi\,\varphi\Big(\frac{\mu}{\sigma}\Big)\left(\sigma-(1-\phi)\frac{\mu^2}{\sigma}\right)=2\frac{\phi}{\sigma}\,\varphi\Big(\frac{\mu}{\sigma}\Big)\left(1-(1-\phi)\,\frac{\mu^2}{\sigma^2}\right)
\end{aligned}
\tag{23}
$$

.

Therefore,

$$
\alpha_z > 0 \iff \phi > 1-\frac{1}{(\mu/\sigma)^2} \tag{24}
$$

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References

Asness, Clifford S., Robert J. Krail, and John M. Liew. 2001. "Do Hedge Funds Hedge?" *Journal of Portfolio Management* 28 (1): 6–19.

Asvanunt, Attakrit, Lars N. Nielsen, and Daniel Villalon. 2015. "Working Your Tail Off: Active Strategies Versus Direct Hedging." *The Journal of Investing* 24 (2): 134–145.

Baltas, Nick, and Desi Ivanova. 2019a. "'Pick Your Battles' & The Challenges of Factor Timing." *Goldman Sachs STS - Insights* (2).

———. 2019b. "How much to allocate to Trend-Following?" *Goldman Sachs STS - Insights* (4).

Baz, Jamil, Josh Davis, Steve Sapra, Jerry Tsai, and Normane Gillmann. 2019. "A Theoretical Framework for Equity-Defensive Strategies." *PIMCO - Quantitative Research* (May).

Christoffersen, Peter F., and Francis X. Diebold. 2006. "Financial Asset Returns, Direction-of-Change Forecasting, and Volatility Dynamics." *Management Science* 52 (8): 1273–1287.

DaSilva, Alexandre S., Wai Lee, and Bobby Pornrojnangkool. 2009. "The Black–Litterman Model for Active Portfolio Management." *Journal of Portfolio Management* 35 (2): 61–70.

Goyal, Amit, and Narasimhan Jegadeesh. 2018. "Cross-Sectional and Time-Series Tests of Return Predictability: What is the Difference?" *Review of Financial Studies* 31 (5): 1784–1824.

Harvey, Campbell R., Edward Hoyle, Sandy Rattray, Matthew Sargaison, Dan Taylor, and Otto Van Hemert. 2019. "The Best of Strategies for the Worst of Times: Can Portfolios Be Crisis Proofed?" *Journal of Portfolio Management* 45 (5): 7–28.

Huang, Dashan, Jiangyuan Li, Liyao Wang, and Guofu Zhou. 2020. "Time Series Momentum: Is it There?" *Journal of Financial Economics, forthcoming.*

Hurst, Brian, Yao Hua Ooi, and Lasse Heje Pedersen. 2017. "A Century of Evidence on Trend-Following Investing." *Journal of Portfolio Management* 44 (1): 15–29.

Kim, Abby Y., Yiuman Tse, and John K. Wald. 2016. "Time Series Momentum and Volatility Scaling." *Journal of Financial Markets* 30:103–124.

Lee, Wai. 2000. *Theory and Methodology of Tactical Asset Allocation.* John Wiley & Sons, New York, NY.

Levine, Ari, and Lasse Heje Pedersen. 2016. "Which Trend Is Your Friend?" *Financial Analysts Journal* 72 (3): 51–66.

Moskowitz, Tobias J., Yao Hua Ooi, and Lasse Heje Pedersen. 2012. “Time Series Momentum.” *Journal of Financial Economics* 104 (1-12): 228–250.

Scholes, Myron, and Joseph Williams. 1977. “Estimating Betas from Nonsynchronous Data.” *Journal of Financial Economics* 5 (3): 309–327.